The geometry of Shimura varieties over $\mathbb{C}$

Yota Maeda
Department of Mathematics, Faculty of Science,
Kyoto University *

"On the occasion of Yota Maeda's 24th birthday"
at Kyoto University，2021 年 3 月

1 Abstract

Shimura varieties are complex projective varieties, which are defined by reductive algebraic groups, Hermitian symmetric domains and arithmetic subgroups, and it is known that they have the models over number fields. Absolute Galois groups act on the Étale cohomology of their models and they are important objects in Arithmetic Geometry since they contribute to the partial solution to the Langlands conjecture. On the other hand, Shimura varieties as complex projective varieties have the structures as moduli spaces. Moreover, there are highly symmetric function, which is called "modular forms", as the sections of Hodge bundles and by using them, one can construct the Baily-Borel compactification of them. Hence Shimura varieties are also interesting objects in Algebraic Geometry.

In this talk, I will introduce the solution to the Kudla's modularity conjecture ([1], [4]), which describes a certain relation between Shimura varieties and modular forms, and their geometric description, that is, the singularities and Kodaira dimension ([2], [3]). They are the work in my Master's course.

References

[1] Maeda, Y., *The Modularity of Special Cycles on Orthogonal Shimura Varieties over Totally Real Fields under the Beilinson–Bloch Conjecture*, Canad. Math. Bull (2020).

[2] Maeda, Y., *The singularities and Kodaira dimension of unitary Shimura varieties*, preprint (2020), arXiv:2008.08095.

*y.maeda@math.kyoto-u.ac.jp

[3] Maeda, Y., *Uniruledness of unitary Shimura varieties associated with Hermitian forms of signatures* $(1,3)$, $(1,4)$ *and* $(1,5)$, preprint (2020), arXiv:2008.13106.

[4] Maeda, Y., *Modularity of special cycles on unitary Shimura varieties over CM-fields*, preprint (2021), arXiv:2101.09232.