Hecke algebras for tame supercuspidal types

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1 Abstract

The representation theory of reductive groups defined over p-adic fields (called
p-adic groups) is interesting because the phenomena are different from the representation theory of real reductive groups. For instance, in the p-adic case, there
are many *supercuspidal representations*. Moreover, there is a conjectural correspondence called *local Langlands correspondence*, which is a correspondence between finite sets of irreducible representations of p-adic groups and L-parameters
of the Weil groups of p-adic fields. Therefore the representation theory of p-adic
groups is also important from the perspective of the number theory.

The category of smooth representations of p-adic groups is decomposed into
the product of full subcategories called *Bernstein blocks*. If there are good pairs
of compact open subgroups and its irreducible representations, which are called
types, each Bernstein block is equivalent to the module category of the Hecke
algebra associated with a type.

In [3], [4] the notion of *depth* of types is defined, and in [2] the Hecke algebras
for depth-zero types, which are the simplest cases, are calculated. In [2], Morris
gives generators and relations for the Hecke algebras. In [5], Yu constructs
types of general depth. His construction starts with depth-zero types of some
twisted Levi subgroups, and he constructs types of more high depth of more large
twisted Levi subgroups inductively. He conjectured that the Hecke algebras
associated with types appearing in each inductive step are all isomorphic [5,
Conjecture 0.2].

In [1], I prove [5, Conjecture 0.2]. In this talk, I will explain the proof of it.

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