

Ahlfors-Beurling v.s. Douady-Earle extension

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1 Abstract

To define a Teichmüller space of a hyperbolic compact Riemann surface, we often use teichmüller equivalent. Let R_j be Riemann surfaces and $f_j : R \rightarrow R_j$ be quasiconformal mappings ($j = 0, 1$), then it is said to (R_1, f_1) is teichmüller equivalent to (R_2, f_2) if there exists a conformal mapping $c : R_1 \rightarrow R_2$ which is homotopic to $f_2 \circ f_1^{-1}$. In classical results, any quasiconformal map $f : \mathbb{H} \rightarrow \mathbb{H}$ has extension to clouser of \mathbb{H} . (Such mapping is equal to quasisymmetric)

Now, the teichmüller equivalent implies that $\tilde{f}_1 = \tilde{f}_2$ on \mathbb{R} , where \tilde{f}_j are lifting of f_j to a universal covering \mathbb{H} , and it is said the circle equivalent. Conversely, the circle equivalent implies the teichmüller equivalent. To prove, it is important to consider of a quasiconformal extension of quasisymmetric map.

In this talk, I will introduce quasiconformality of two extensions : Ahlfors-Beurling extension([1]) and Douady-Earle extension([2] and [3]), and my idea that relationship between the extensions with respect to “Normalization and Compactness”.

References

- [1] Lars V. Ahlfors, C.J.Earle, I.Kra, M. Shishikura, J.H. Hubbard “Lectures on Quasiconformal Mappings” Second Edition (American Mathematical Society, 2006)
- [2] Frederick.P.Gardiner and Nilola Lakic “Quasiconformal Teichmüller Theory” (American Mathematical Society, 2000)
- [3] Jun Hu and Oleg Muzician “Cross-ratio distortion and Douady-Earle extension: I. A new upper bound on quasiconformality” London Math. Soc(2) 86 (2012) 387-406

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