On the distrbution of sums of two squares

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"On the occasion of Yota Maeda's 24th birthday"
at Kyoto University

1 Abstract

A sum of two square is literally the number which can be represented as the sum of two squares of integers. The neccesary and sufficient condition that a certain number may be represennted so has already been found by Euler in 18c using the factorization of it[1], and the global asymptotic estimation of sums of two squares was obtained by Landau[2] in 1907. On the other hand, the consequences about "local" distributions of the sums were gained with more contemporary tools. In 1953 Pracher[3] gave the distribution of sums of two squares in arithmetic progressions asymptotically, and Iwaniec[4] showed in 1976 the estimation with explicit error term beyond the restriction about the size of modulus of arithmetic progressions. For the distribution in short intervals, Hooley[5] proved in 1994 that the actual number of the sums belonging to almost all short interval $[x, x + \log x]$ had the same order of magnitude with "naive" supposition coming from the work of Landau, but not for all the intervals by Balog-Wooley[6] in 2000.

In this talk, I will explain the works of Iwaniec and Balog-Wooley especially, and they both are considerablly based on the theory called "sieve method", so I will also introduce the method briefly.

References

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